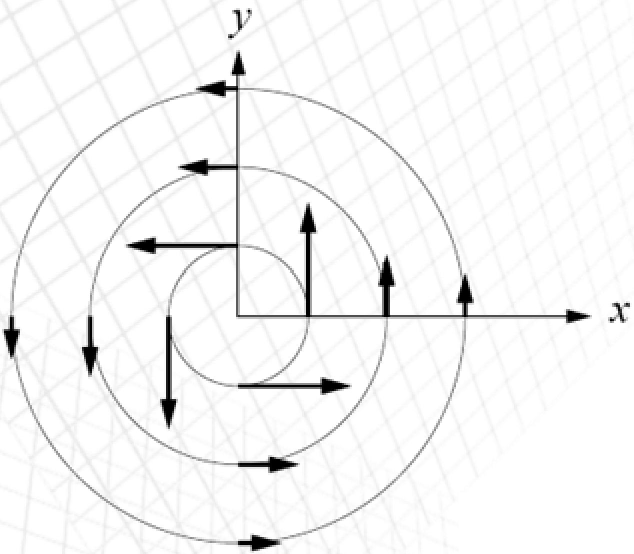
< 3.14. Vortex Flow >



By definition,

$$V_\theta = \frac{const}{r}, \ V_r = 0$$

$$\Gamma = \oint \vec{V} \bullet d\vec{s} = - V_\theta \bullet (2\pi r)$$

➔ $V_\theta = - \dfrac{\Gamma}{2\pi r}$

➔ $\psi = \dfrac{\Gamma}{2\pi} ln r$

➔ $\phi = - \dfrac{\Gamma}{2\pi} \theta$

(NOTE)
1. Vortex flow is irrotational except its origin
2. Circulation is positive-clockwise

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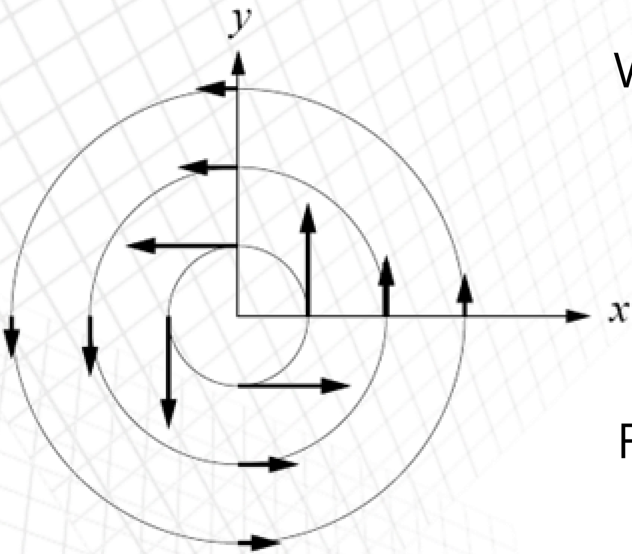


What happens at r=0 ?

$$\Gamma = -\iint_S (\nabla \times \vec{V}) d\vec{S}$$

$$2\pi C = \iint_S (\nabla \times \vec{V}) d\vec{S} \quad \because C = -\frac{\Gamma}{2\pi}$$

For 2D flow

$$2\pi C = \iint_S |\nabla \times \vec{V}| dS$$

Γ is the same for all the circulation streamlines

$$\lim_{dS \to 0} \iint_S |\nabla \times \vec{V}| dS = |\nabla \times \vec{V}| dS \;\rightarrow\; 2\pi C = |\nabla \times \vec{V}| dS$$

As $r \to 0 \;\; ds \to 0 \quad \rightarrow \quad \nabla \times \vec{V} \to \infty$

< 3.15. Lifting Flow over a Cylinder >

❖ Uniform flow + doublet + vortex

$$\psi = (V_\infty r sin\theta)(1 - \frac{R^2}{r^2}) + \frac{\Gamma}{2\pi} ln \frac{r}{R}$$

$$\frac{1}{r}\frac{\partial\psi}{\partial\theta} = V_r = (1 - \frac{R^2}{r^2})\, V_\infty \cos\theta$$

$$-\frac{\partial\psi}{\partial r} = V_\theta = -(1 + \frac{R^2}{r^2})\, V_\infty \sin\theta - \frac{\Gamma}{2\pi r}$$

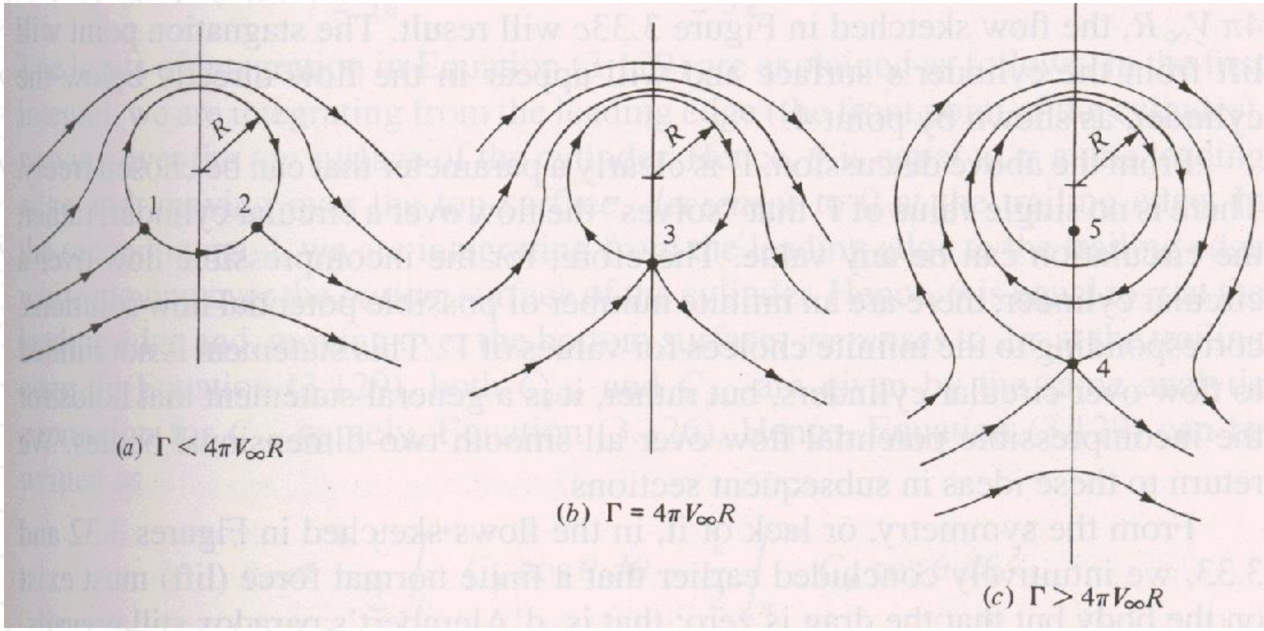
< 3.15. Lifting Flow over a Cylinder >

❖ Stagnation point

$$V_r = V_\theta = 0$$

$$(r, \theta) = \left(R, \sin^{-1}\left(-\frac{\Gamma}{4\pi V_\infty R}\right) \right)$$

Not valid for third case



(a) $\Gamma < 4\pi V_\infty R$

(b) $\Gamma = 4\pi V_\infty R$

(c) $\Gamma > 4\pi V_\infty R$

< 3.15. Lifting Flow over a Cylinder >

At surface,

$$V_\theta = -2V_\infty \sin\theta - \frac{\Gamma}{2\pi R}$$

$$\rightarrow C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - \left(4\sin^2\theta + \frac{2\Gamma\sin\theta}{\pi R V_\infty} + \left(\frac{\Gamma}{2\pi R V_\infty}\right)^2\right)$$

$$c_d = \frac{1}{c}\int_{LE}^{TE} (C_{p,u} - C_{p,l})\,dy \quad \rightarrow \quad c_d = 0$$

→ The drag on a cylinder is zero, regardless of whether or not having circulation in inviscid, irrotational and incompressible flow.

< 3.15. Lifting Flow over a Cylinder >

Put Cp expression into this integral

$$c_l = \int_0^c (C_{p,l} - C_{p,u})dx \quad \rightarrow \quad c_l = \frac{\Gamma}{RV_\infty}$$

Sectional Lift, $L' = q_\infty S c_l$

$$= \frac{1}{2}\rho V_\infty^2 S c_l = \frac{1}{2}\rho V_\infty^2 S \frac{\Gamma}{RV_\infty}$$

$$= \rho V_\infty \Gamma$$

$(S = 2R : planform\ area)$

$$\therefore L' = \rho V_\infty \Gamma$$

"Kutta-Joukowski Theorem"